Calculus Warm Up # 7-1

Sketch the region bounded by the graphs and then find the area.

1. $y = 1 - \frac{1}{x}$, $y = x - 2$

2. $y = \frac{1}{x^2}$, $y = 4$, $x = 5$

Tam Review Answers:

1. $\frac{9}{2}$
2. $\frac{48\pi}{5}$
3. $\frac{1178\pi}{105}$
4. $\frac{104\pi}{15}$
5. $\frac{15\pi}{2}$
6. $\frac{64\pi}{5}$
7. 7500 in-lbs
8. 7020π ft-lbs.
9. a. ≈ 6.38
   b. ≈ 10.50
10. $\frac{\pi}{27}(145^{3/2} - 1)$
    ≈ 203.04
11. ≈ 162.55

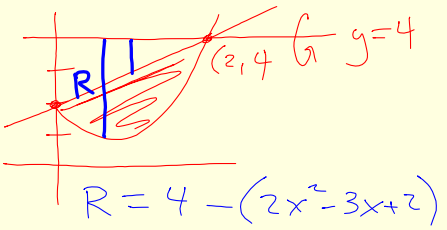
HW Questions: tan ws

AP Calculus
Ch. 6 Review

Name _____

1. Determine the area bounded by $y = x^2 - 2x + 1$ and $y = x + 1$. Evaluate the integral by hand.

2. Write the definite integral that represents the volume of the solid formed by revolving the region bounded by $y = 2x^2 - 3x + 2$ and $y = x + 2$ about the line $y = 4$. Use your calculator to evaluate the integral.



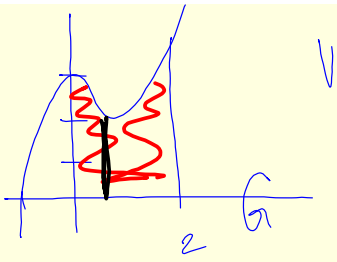
$$\pi \int_0^2 \left[(-2x^2 + 3x + 2)^2 - (-x+2)^2 \right] dx$$

$R = 4 - (2x^2 - 3x + 2)$

$r = 4 - (x+2)$

$= -x + 2$

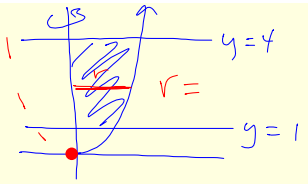
3. Write the definite integral that represents the volume of the solid formed by revolving the region bounded by $y = x^3 - 2x^2 + 3$, $x = 0$, $x = 2$, and $y = 0$ about the x-axis. Use your calculator to evaluate the integral.



$$V = \pi \int_0^2 (\quad)^2 \, dx$$

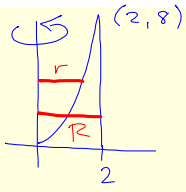
4. Find the volume of the solid formed by revolving the region bounded by $y = x^2 - 2x + 3$ and $y = 3$ about the x-axis. Use your calculator to evaluate the integral.

5. Find the volume of the solid formed by revolving the region bounded by $y = x^2$, $y = 1$, and $y = 4$ about the y-axis.



6. Find the volume generated by revolving the area bounded by $y = x^3$, $y = 0$, and $x = 2$ about the y-axis.

use washers, then do it again with shells.



$R = 2$

$r = \sqrt[3]{y}$

$x = \sqrt[3]{y}$

$$V = \pi \int_0^8 \left[2^2 - \left(\sqrt[3]{y}\right)^2\right] dy$$

$$V = \pi \int_0^8 \left(4 - y^{2/3}\right) dy$$

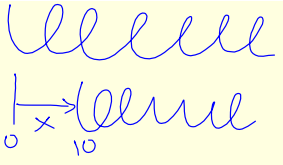
$$= \pi \left[4y - \frac{3y^{5/3}}{5}\right]_0^8$$

$$= \pi \left(32 - \frac{3(32)}{5} - 0\right)$$

7. A force of 600 pounds compresses a spring 4 inches from its natural length. Find the work done in compressing the spring 10 inches from its natural length. (Give your answer in inch-pounds).

$600 = k(4)$

$k = \frac{150 \text{ lbs}}{\text{in}}$

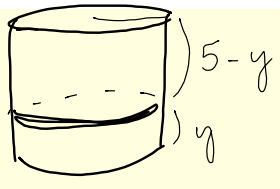


$F(x) = 150x$

$$W = \int_0^{10} 150x \, dx$$

8. A cylindrical tank of radius 3 feet is 5 feet high and full of water. How much work is done pumping out all the water through a hole in the top of the tank? (Assume the weight of the water to be 62.4 pounds per cubic foot).

$r = 3$



$V = 9\pi \, \Delta y$

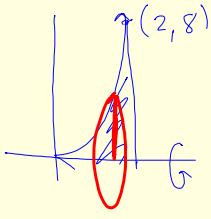
$F = 62.4(9\pi \, \Delta y)$

$$W = \int_0^5 (5 - y)(561.6\pi) \, dy$$

9. Write and simplify the integral, then calculate the arc length of the curve on the given interval. (2 dec. places)

a) $y = x^2 - 3x + 4$, $[2,4]$ b) $y = \frac{2}{3}x^2 + 5$, $[1,4]$

10. Find, (by hand), the surface area of the solid created by revolving $y = x^3$ about the x-axis on $[0,2]$.



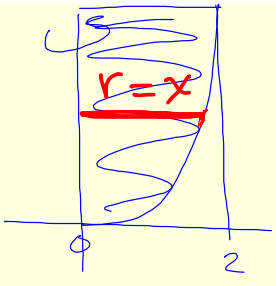
$$S = 2\pi \int_0^2 x^3 \sqrt{1 + (3x^2)^2}\, dx$$

$$S = 2\pi \frac{1}{36} \int_0^2 36x^3 \sqrt{1 + 9x^4}\, dx$$

$u = 1 + 9x^4$

$du = 36x^3\, dx$

11. Write the integral and calculate the surface area of the solid created by revolving the region bounded by $y = x^4$, $x = 0$, and $x = 2$ about the y-axis.



$$S = 2\pi \int_0^2 x \sqrt{1 + (4x^3)^2}\, dx$$

$$= 2\pi \int_0^2 x \sqrt{1 + 16x^6}\, dx$$

6.6 Fluid Pressure

Force Exerted by a Fluid

Last topic in Chapter 6.

Test this Thursday

Definition of Fluid Pressure

$$p = wh$$

p = fluid pressure: The force per unit of area

w = weight density: The weight of the liquid per unit of volume

h = depth from the surface

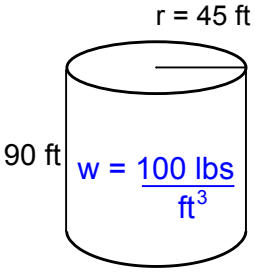
Since w is constant, the pressure depends entirely on depth from the surface!

The Great Molasses flood of 1917

The molasses tank in Boston exploded and flooded the streets 30 feet deep!

What was the force, F, of the molasses on the base of the tank?

$p = wh$



$p = \left(\frac{100\ lbs}{ft^2}\right)(90\ ft)$

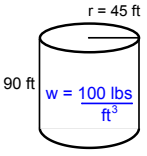
$P = \frac{9000\ lbs}{ft^2}$

F = (pressure)(area)

$= \frac{9000\ lbs}{ft^2} \cdot \pi(45)^2\ ft^2$

$\approx 57,225,526$ lbs.

What was the total force against the bottom foot of the tank wall?



bottom 1 foot:

y = depth of a representative slice

Area of the unwrapped slice: $(2\pi r)(\Delta y)$

$$\text{Total Force} = \int_{89}^{90} (\text{pressure})(\text{area})$$

$$= \int_{89}^{90} \left(\frac{100\ lbs}{ft^2}\right)(y\ ft)(2\pi(45))\,dy$$

$$9000\pi \int_{89}^{90} y\,dy$$

$\approx 2,530,553$ lbs.

For a Submerged Vertical Plane

* Definition of Force exerted by a fluid over a depth of $y = [c, d]$

$$F = \int_c^d (\text{pressure})(\text{area of a slice})$$

$$l \cdot w$$

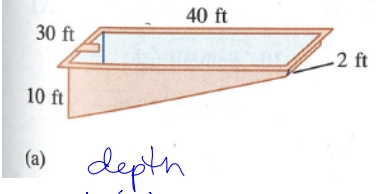
$$(w \cdot h(y))(L(y) \cdot w)$$

$L(y)$ = horizontal length at depth y.

$h(y)$ = depth in terms of y.

EXAMPLE 4 Fluid force on a vertical surface

A swimming pool is 2 feet deep at one end and 10 feet deep at the other, as shown in Figure 6.59(a). The pool is 40 feet long and 30 feet wide with vertical sides. Find the fluid force against one of the 40-foot sides.



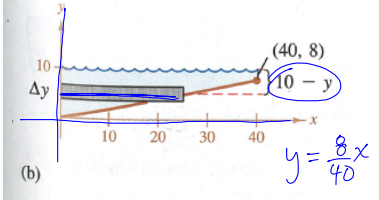


FIGURE 6.59

depth

$h(y) = 10 - y$

$y = \frac{8}{40}x$

$$F = \int \left(\frac{62.4 \text{ lbs}}{\text{ft}^3} \cdot (10 - y)\right)(L(y)\,dy)$$

pressure · area

$$F = 62.4\left[\int_0^8 (10 - y)(5y)\,dy + \int_8^{10} (10 - y)(40)\,dy\right]$$

HW: p. 342

1 - 17 odd

Ch. 6 Test: Thursday

HW Quiz tomorrow:

pgs. 327, 328, 335, tan review